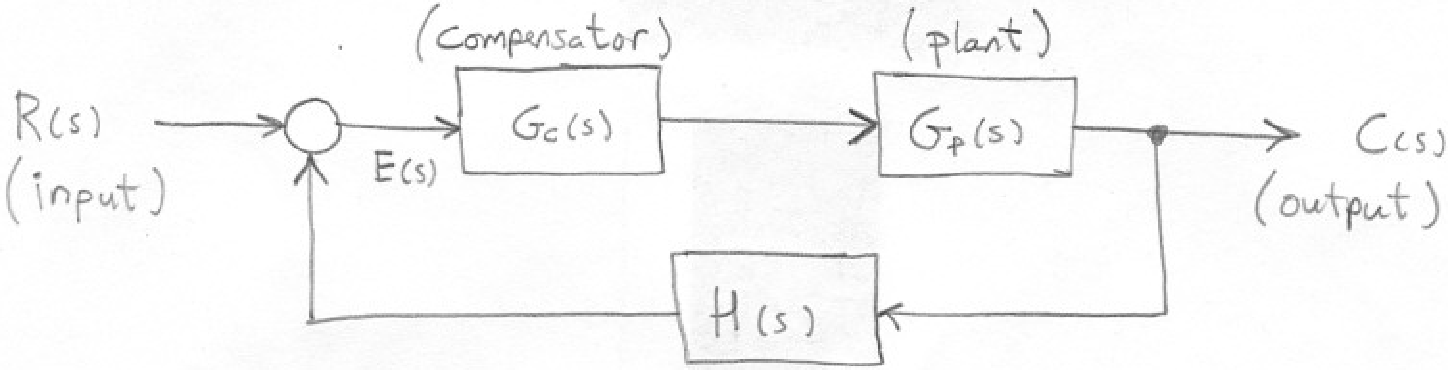
Last lecture we saw how a PID controller can change the dynamics of a first order plant. Now let's design a PID controller for a second order plant. Recall from p 4-3 the overall system



The typical second order plant has the following form:

$$G_p(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad \text{(open loop system)}$$

On p. 4-4 we derived the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G_c G_p}{1 + G_c G_p H} \qquad \text{(closed loop system) (1)}$$

Depending on the type of compensator you choose, the closed loop system dynamics will be affected. For a PID controller

$$G_c(s) = K_p + \frac{K_I}{s} + sK_D$$

Depending on the controller we choose P, PI, PD, PID we can shape the system response.

Proportional Control (unity feedback)

$$G_c(s) = K_p \rightarrow \text{substitute into Eq. 1}$$

(2) $$\frac{C(s)}{R(s)} = \frac{K_p\left(\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right)}{1+K_p\left(\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right)} = \frac{K_p\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2(1+K_p)}$$

By changing K_p we can move the pole locations and affect the response

Proportional Derivative Control (unity feedback)

$$G_c(s) = K_p + K_D s \rightarrow \text{substitute into Eq. 1}$$

$$\frac{C(s)}{R(s)} = \frac{(K_p+K_D s)\left(\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right)}{1+(K_p+K_D s)\left(\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{(K_p+K_D s)\omega_n^2}{s^2+(K_D\omega_n^2+2\zeta\omega_n)s+\omega_n^2(K_p+1)} \quad (3)$$

This term is associated with the system damping and so as K_D increases the system damping will increase. Unfortunately the numerator contains a term that, loosely speaking, will act like a high pass filter and amplify high frequency noise.

Proportional Integral Control (unity feedback)

$$G_c(s) = K_P + \frac{K_I}{s} \rightarrow \text{substitute into Eq. 1}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s}(sK_P + K_I)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)}{1 + \frac{1}{s}(sK_P + K_I)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{(sK_P + K_I)\,\omega_n^2}{s^3 + 2\zeta\omega_n s^2 + (K_P + 1)\omega_n^2 s + K_I\omega_n^2} \qquad (4)$$

The system order has been increased to third order. As for first order plants, the integral component will improve the steady state error.

Proportional Integral Derivative Control, PID (unity feedback)

$$G_c(s) = K_P + \frac{K_I}{s} + K_D s \rightarrow \text{substitute into Eq. 1}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s}(s^2 K_D + sK_P + K_I)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)}{1 + \frac{1}{s}(s^2 K_D + sK_P + K_I)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{(s^2 K_D + sK_P + K_I)\,\omega_n^2}{s^3 + (2\zeta\omega_n + K_D\omega_n^2)s^2 + (K_P + 1)\omega_n^2 s + K_I\omega_n^2} \qquad (5)$$

If we let $T_i = \frac{K_I}{K_P}$ and $T_d = \frac{K_D}{K_P}$

and substitute into Eq. 5 we can express the transfer function in terms of a single variable, K_p (assuming T_i and T_d are constant)

$$(6) \qquad \frac{C(s)}{R(s)} = \frac{\omega_n^2 K_p \left(T_d s^2 + s + T_i \right)}{s^3 + \left(2\zeta\omega_n + T_d K_p \omega_n^2\right) s + (K_p+1)\omega_n^2 s + K_p T_i \omega_n^2}$$

- Choosing the values for K_P, K_I, and K_D or K_P, T_i, and T_d are not intuitively obvious. Methods to do so will be discussed later.

- Some important MATLAB Functions:

TF Creation of transfer functions or conversion to transfer function.

Creation:
SYS = TF(NUM,DEN) creates a continuous-time transfer function SYS with numerator(s) NUM and denominator(s) DEN. The output SYS is a TF object.

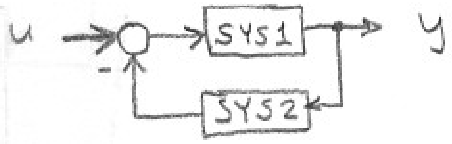
STEP Step response of LTI models.

STEP(SYS,TFINAL) simulates the step response from t=0 to the final time t=TFINAL. For discrete-time models with unspecified sampling time, TFINAL is interpreted as the number of samples.

STEP(SYS,T) uses the user-supplied time vector T for simulation. For discrete-time models, T should be of the form Ti:Ts:Tf where Ts is the sample time. For continuous-time models, T should be of the form Ti:dt:Tf where dt will become the sample time for the discrete approximation to the continuous system. The step input is always assumed to start at t=0 (regardless of Ti).

FEEDBACK Feedback connection of two LTI models.

SYS = FEEDBACK(SYS1,SYS2) computes an LTI model SYS for the closed-loop feedback system



y = SYS * u

$$G_p = \frac{1}{Ts + 1}$$ First Order System

$T = 0.05$

The system response is too slow! *Let's speed it up by using feedback!*

$G_c = K_p + K_i/s \; K_d\, s$

$K_p = 50$; $K_i = 100$; $K_d = 0.1$

Open Loop Transfer Function (Plant):

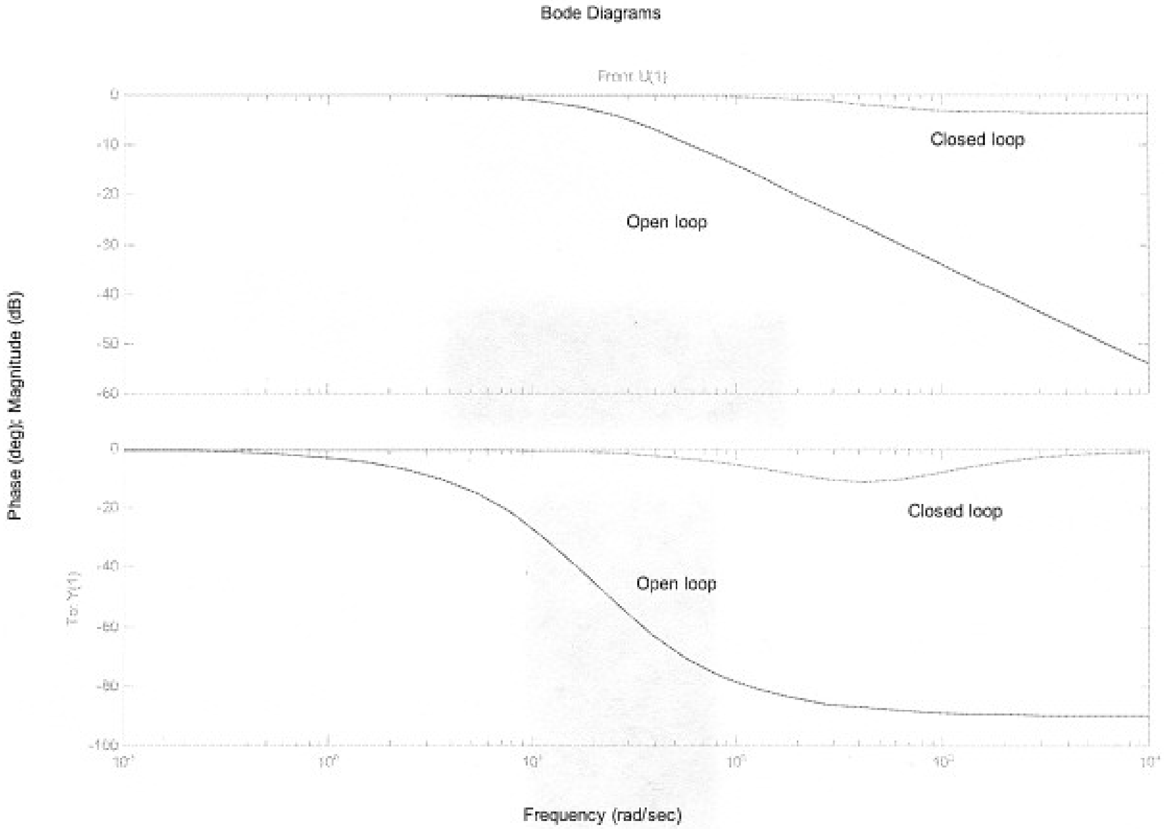
$$G_p = \frac{1}{0.05\, s + 1}$$

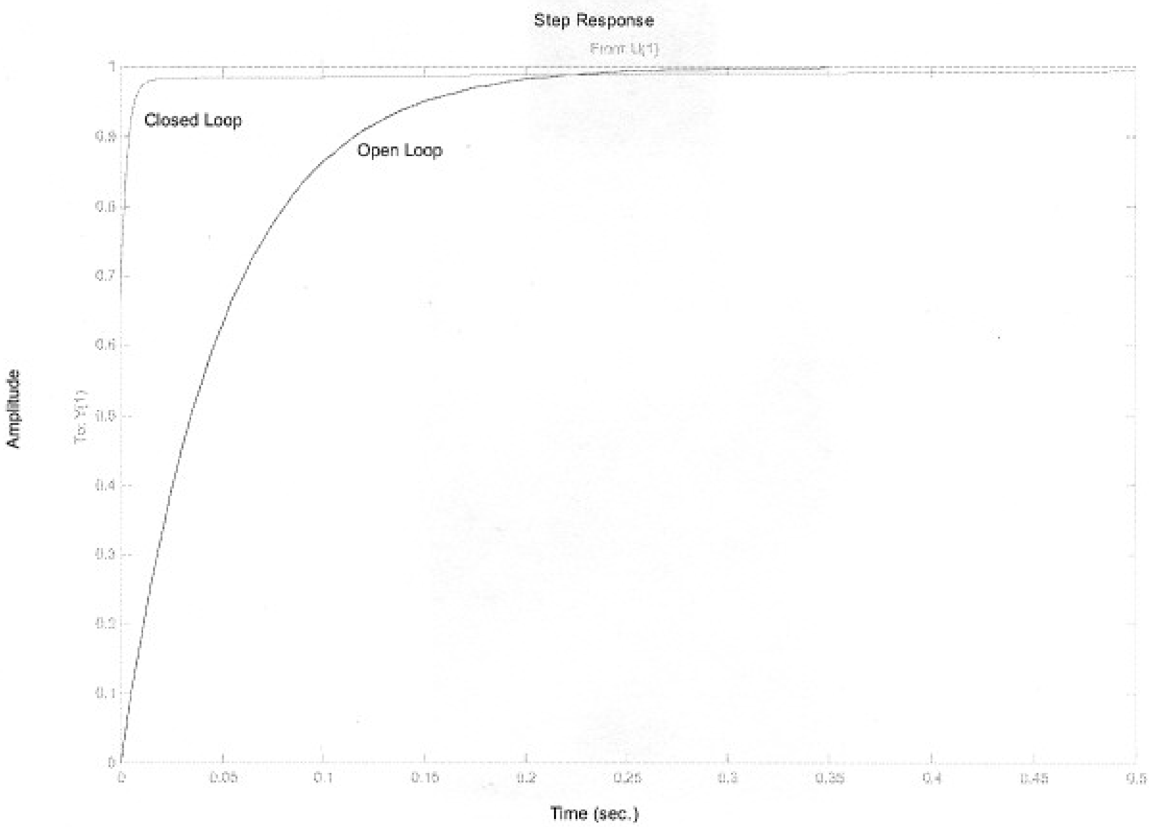
Compensator Transfer function:

$$G_c = \frac{0.1\, s^2 + 50\, s + 100}{s}$$

Closed Loop Transfer function:

$$G_{cl} = \frac{0.1\, s^2 + 50\, s + 100}{0.15\, s^2 + 51\, s + 100}$$


Bode Diagrams
From: U(1)
Closed loop
Open loop
Closed loop
Open loop
Phase (deg); Magnitude (dB)
To: Y(1)
0
-10
-20
-30
-40
-50
-60
0
-20
-40
-60
-80
-100
Frequency (rad/sec)


Step Response
From: U(1)
Closed Loop
Open Loop
Amplitude
To: Y(1)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Time (sec.)